# Re-evaluating the Triple-Colocation Analysis for Estimating Aquarius Satellite Salinity Error

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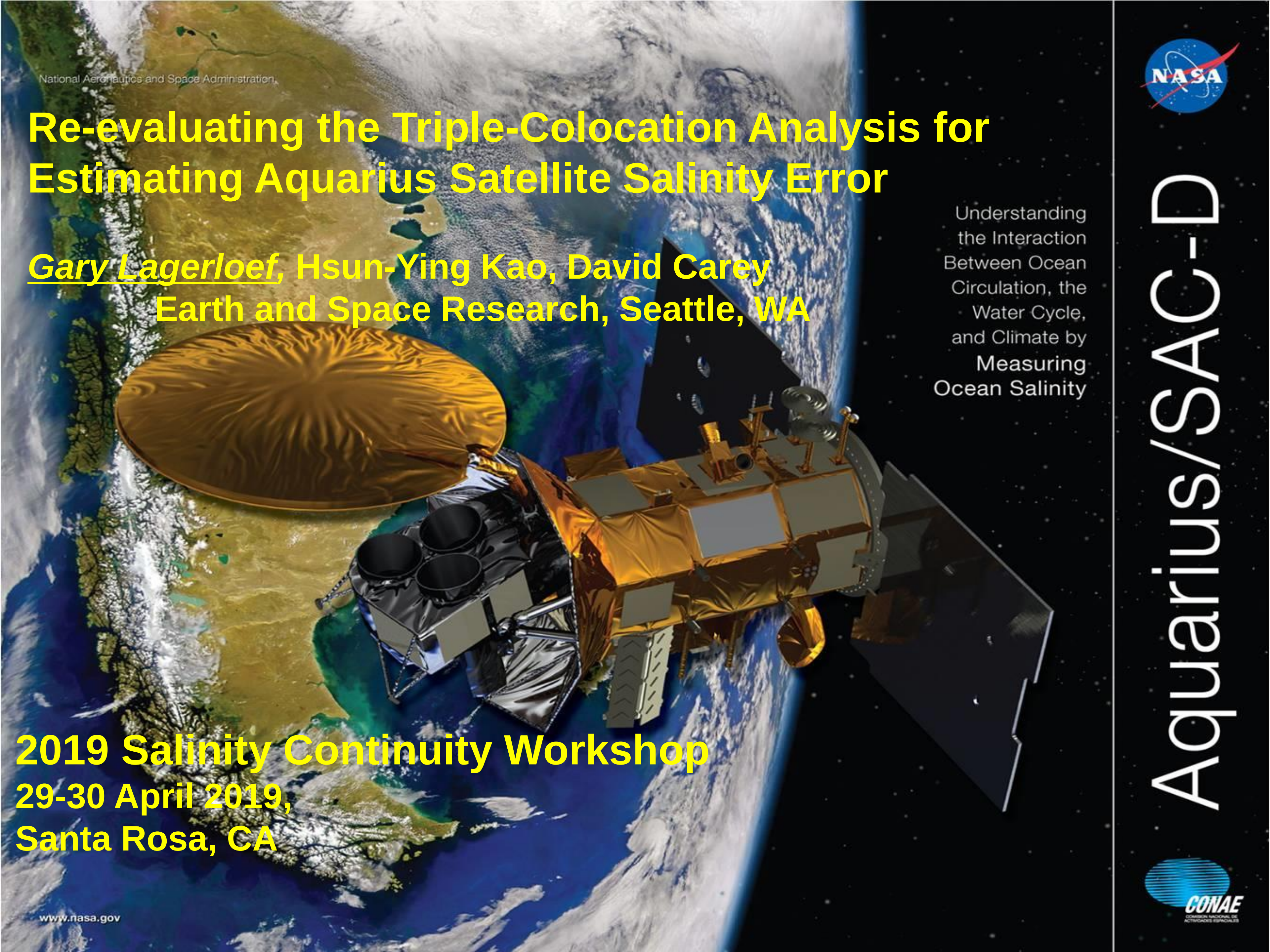
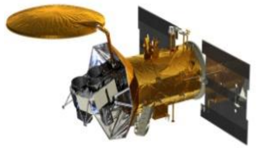


# Outline



- Triple-point Analysis (TPA): An *error variance calculation* based on a set of simultaneous co-located measurements from three different systems (e.g. satellite, in situ and numerical analysis). [Stoffelen, 1998]

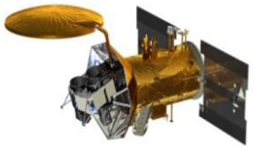
- For Aquarius, we have used a variation of Stoffelen's approach.

- Each approach derives a unique error variance of each of the three measurements.

- Each also assumes that the errors of each measurement system are *un-correlated* with one-another.

- Here, we seek to verify if this assumption is true; and consider the consequences if not.

# Statement of the Problem



**(1)**

$$S_1 = ST + \mathcal{E}_1$$
$$S_2 = ST + \mathcal{E}_2$$
$$S_3 = ST + \mathcal{E}_3$$

Aquarius L2 data co-located data:

1) Aquarius (AQ)
2) In Situ (IS)
3) Hycom (HY)

$S_T$ is true SSS

**$\mathcal{E}_n$ is the measurement error for n**

**(2)**

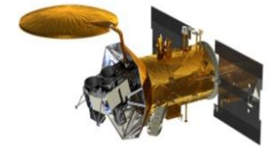
$$S_1 - S_2 = \mathcal{E}_1 - \mathcal{E}_2$$
$$S_2 - S_3 = \mathcal{E}_2 - \mathcal{E}_3$$
$$S_1 - S_3 = \mathcal{E}_1 - \mathcal{E}_3$$

$$\Delta S_{12} = \Delta\mathcal{E}_{12}$$
$$\Delta S_{23} = \Delta\mathcal{E}_{23}$$
$$\Delta S_{13} = \Delta\mathcal{E}_{13}$$

$$< \Delta S_{ij}{}^2 > = < \mathcal{E}_i{}^2 > + < \mathcal{E}_j{}^2 >$$

$< \mathcal{E}_{ij}{}^2 > = 0$ for i≠j

Assume error cross-correlations are zero



$$< \Delta S_{12}^2 > = < \varepsilon_1^2 > + < \varepsilon_2^2 >$$
$$< \Delta S_{23}^2 > = < \varepsilon_2^2 > + < \varepsilon_3^2 >$$
$$< \Delta S_{13}^2 > = < \varepsilon_1^2 > + < \varepsilon_3^2 >$$

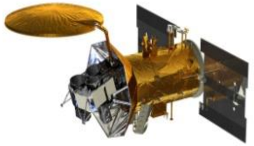
Closed set of 3 equations with 3 unknowns $< \varepsilon_1^2 >, < \varepsilon_2^2 >, < \varepsilon_3^2 >$

$$< \varepsilon_{12} > = < \varepsilon_{23} > = < \varepsilon_{13} > = 0$$

$$< \varepsilon_1^2 > = (< \Delta S_{12}^2 > + < \Delta S_{13}^2 > - < \Delta S_{23}^2 >)/2$$
$$< \varepsilon_2^2 > = (< \Delta S_{12}^2 > + < \Delta S_{23}^2 > - < \Delta S_{13}^2 >)/2$$
$$< \varepsilon_3^2 > = (< \Delta S_{13}^2 > + < \Delta S_{23}^2 > - < \Delta S_{12}^2 >)/2$$

3 equations to compute $< \varepsilon_i^2 >$ from the observed $< \Delta S_{ij}^2 >$.

These are the equations that we have used for *Aquarius* triple-point error estimates, Assuming error cross-correlations are negligible.



## Revised Triple-Point analysis equations:

- Without correlated errors (as we have been using).
- Additional error correlation terms

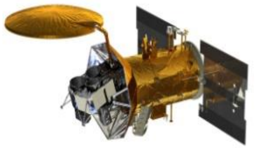
$$< \varepsilon_1^2 > = (< \Delta S_{12}^2 > + < \Delta S_{13}^2 > - < \Delta S_{23}^2 >)/2 + < \varepsilon_{12} > + < \varepsilon_{13} > - < \varepsilon_{23} >$$

$$< \varepsilon_2^2 > = (< \Delta S_{12}^2 > + < \Delta S_{23}^2 > - < \Delta S_{13}^2 >)/2 + < \varepsilon_{12} > + < \varepsilon_{23} > - < \varepsilon_{13} >$$

$$< \varepsilon_3^2 > = (< \Delta S_{13}^2 > + < \Delta S_{23}^2 > - < \Delta S_{12}^2 >)/2 + < \varepsilon_{13} > + < \varepsilon_{23} > - < \varepsilon_{12} >$$

AQUARIUS/SAC-D

# Regression Analysis



**Theory**

Data Input
**DI**
AQ IS HY

Match-up Sequence

1) Aquarius (AQ)
2) In Situ (IS)
3) Hycom (HY)

$S_T$ is true SSS

$$S_1 = ST + \mathcal{E}_1$$
$$S_2 = ST + \mathcal{E}_2$$
$$S_3 = ST + \mathcal{E}_3$$

**Difference**

Data Difference
**DD**
AQ-IS IS-HY

Match-up Sequence

$$S_1 - S_2 = \mathcal{E}_1 - \mathcal{E}_2$$
$$S_2 - S_3 = \mathcal{E}_2 - \mathcal{E}_3$$

Matrix **DD** lacks any information of $S_T$

$S_T$ is the “common signal” to be resolved by regression analysis (next slide).

$\mathcal{E}_n = S_n - S_T$ is the error of the $n^{th}$ measurement system.

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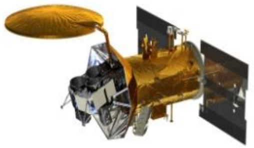
# Test Case and Summary



One day co-location data set;  1 Sep 2011
V5.0 Level 2 data

Correlation Matrix

| | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ |
|---|---|---|---|
| $\varepsilon_1$ | 1.0000 | 0.2394 | 0.2710 |
| $\varepsilon_2$ | 0.2394 | 1.0000 | 0.9995 |
| $\varepsilon_3$ | 0.2710 | 0.9995 | 1.0000 |

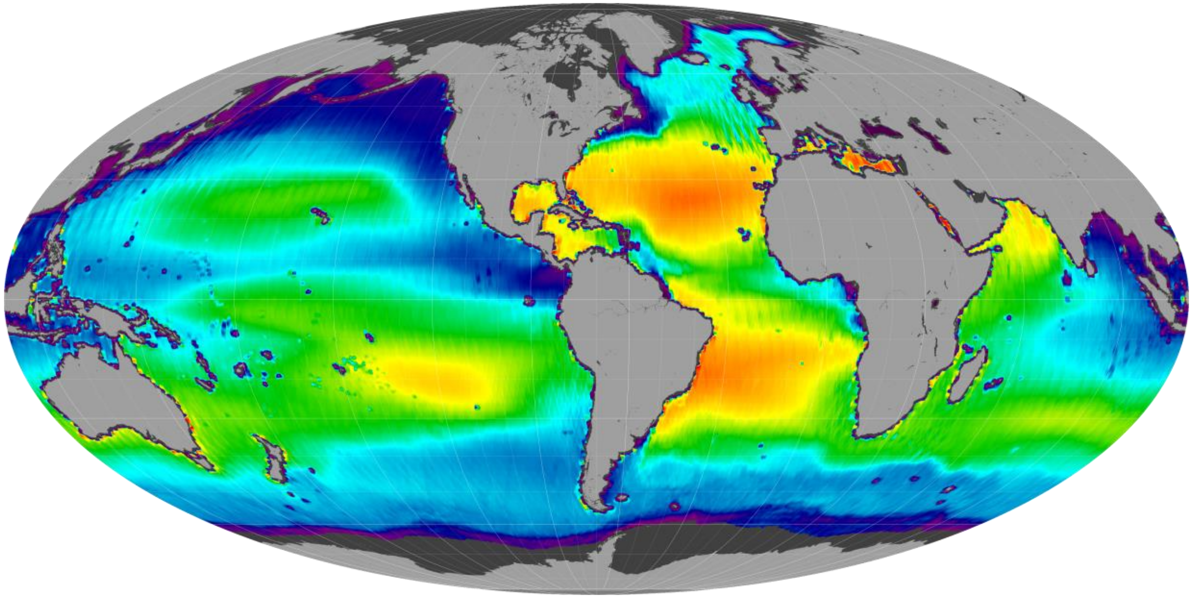
1) Aquarius (AQ)
2) In Situ (IS)
3) Hycom (HY)

- ***Yes, the cross correlations are definitely non-zero.***
- Future work:
  - Evaluate the effect on the derived error variances
  - More extensive study of Level 2 data
  - Biases in the three co-located measurements were removed and the effect needs to be studied more extensively
  - Apply the analysis to the monthly co-located analysis used to derive the monthly errors in the V5.0 Validation analysis document, and revise results as needed.

# Final Comments

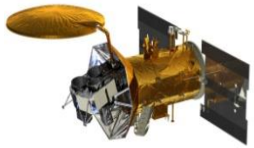
- Although the Aquarius mission is over, it has provided a legacy data set from which there is still much to learn.
- Aquarius data lives!

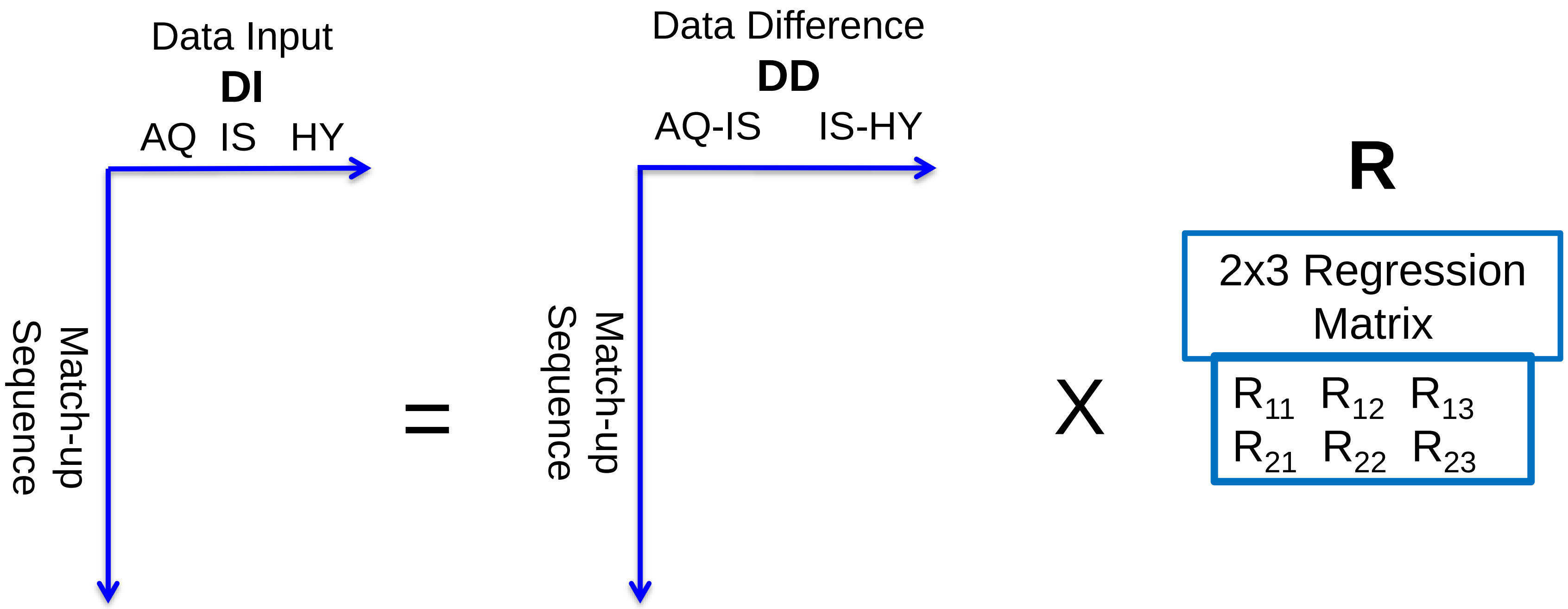
V5.0 Mean



# Backup Material

# Regression Analysis






1. Regression: **R** = **DD\ DI**
2. Inverse: $\mathbf{DI_r} = \mathbf{DD*R}$ is expected to contain measurement error ($\mathcal{E}_n$) but not $S_T$
3. $\mathbf{DI - DI_r = S_T}$ Solves for $S_T$
4. $\mathcal{E}_n = \mathbf{DI_r}$

# How to estimate uncertainty of Aquarius and validation data

The satellite salinity measurement $S_S$ and the *in situ* validation measurement $S_V$ are defined by:

$$S_S = S \pm \varepsilon_S$$

$$S_V = S \pm \varepsilon_V$$

where S is the true surface salinity averaged over the Aquarius footprint area and microwave optical depth in sea water (~ 1 cm). $\varepsilon_S$ and $\varepsilon_V$ are the respective satellite and *in situ* measurement errors relative to S. The mean square of the difference $\Delta S$ between $S_S$ and $S_V$ is given by:

$$\langle \Delta S_{SV}^2 \rangle = \langle \varepsilon_S^2 \rangle + \langle \varepsilon_V^2 \rangle \quad (1)$$

where < > denotes the average over a given set of paired satellite and *in situ* measurements, and $\langle \varepsilon_S \varepsilon_V \rangle = 0$.

Likewise, define HyCOM salinity interpolated to the satellite footprint as $S_H = S \pm \varepsilon_H$, and mean square differences

$$\langle \Delta S_{HV}^2 \rangle = \langle \varepsilon_H^2 \rangle + \langle \varepsilon_V^2 \rangle \quad (2)$$ HyCOM vs in situ validation data

$$\langle \Delta S_{SH}^2 \rangle = \langle \varepsilon_S^2 \rangle + \langle \varepsilon_H^2 \rangle \quad (3)$$ Satellite vs HyCOM

Equations (1)-(3) comprise three equations with three variables given by:

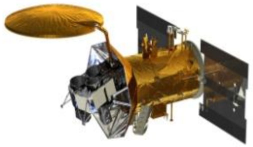
$$\langle \varepsilon_S^2 \rangle = \{\langle \Delta S_{SV}^2 \rangle + \langle \Delta S_{SH}^2 \rangle - \langle \Delta S_{HV}^2 \rangle\} /2 \quad (4)$$ satellite measurement error

$$\langle \varepsilon_H^2 \rangle = \{\langle \Delta S_{SH}^2 \rangle + \langle \Delta S_{HV}^2 \rangle - \langle \Delta S_{SV}^2 \rangle\} /2 \quad (5)$$ HyCOM measurement error

$$\langle \varepsilon_V^2 \rangle = \{\langle \Delta S_{SV}^2 \rangle + \langle \Delta S_{HV}^2 \rangle - \langle \Delta S_{SH}^2 \rangle\} /2 \quad (6)$$ In situ validation measurement error

AQUARIUS/SAC-D

# Triple Point Analysis



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## Toward the true near-surface wind speed: Error modeling and calibration using triple collocation

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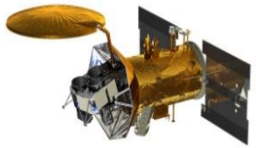

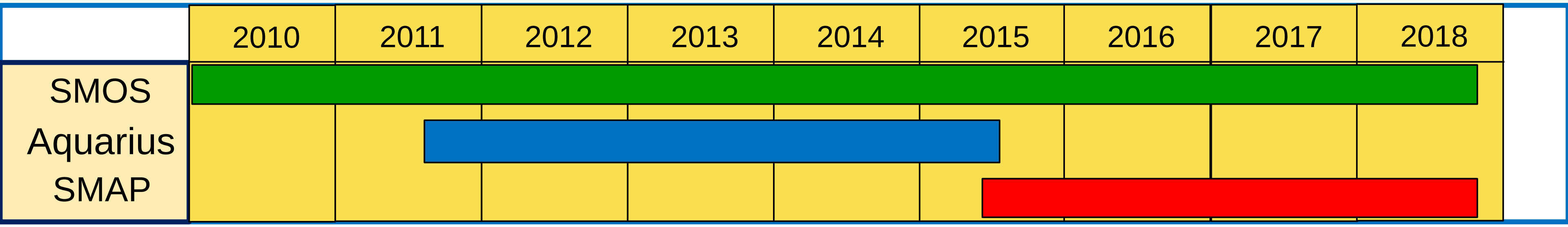


# In Situ Ocean Surface Data